Calculus 1 Workbook Solutions

Other functions and trigonometry

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QUADRATIC FORMULA

1. Solve for x using the quadratic formula.

$$4x^2 - 8x - 15 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-15)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 + 240}}{8}$$

$$x = \frac{8 \pm \sqrt{304}}{8}$$

$$x = \frac{8 \pm 4\sqrt{19}}{8}$$

$$x = \frac{2 \pm \sqrt{19}}{2}$$

2. Write the quadratic formula for the following quadratic equation.

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$$x^2 - 5x - 24 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

We could continue to simplify to solve for the roots.

$$x = \frac{5 \pm \sqrt{25 + 96}}{2}$$

$$5 \pm \sqrt{121}$$

$$x = \frac{5 \pm 11}{2}$$

$$x = -3, 8$$

3. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

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Solution:

The $-b$ at the beginning of the quadratic formula is written as -5, but $b = -5$. Which means it should be written as $-(-5)$.

■ 4. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

Solution:

Rewrite the expression as

$$z^2 = z + 3$$

$$z^2 - z - 3 = 0$$

Then the quadratic formula gives

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{13}}{2}$$

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5. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$____x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(-5)}}{2(-2)}$$

Solution:

The blank should be the term -2.

6. Simplify the expression.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

Solution:

The expression is simplified as

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

$$\frac{8 \pm \sqrt{64 - 56}}{2}$$

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$$\frac{8 \pm \sqrt{8}}{2}$$

$$\frac{8 \pm 2\sqrt{2}}{2}$$

$$4 \pm \sqrt{2}$$

■ 7. What are two ways to solve a quadratic equation when you cannot easily factor?

Solution:

You can either use the method of completing the square or the quadratic formula.

■ 8. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(7)}}{2(1)}$$

Solution:

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The expression was not written in the correct form before using the quadratic formula. It should be written as $x^2 + 2x - 7 = 0$, for which the quadratic formula would then be

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

■ 9. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

Solution:

Rewrite the expression as

$$4t^2 - 1 = -8t$$

$$4t^2 + 8t - 1 = 0$$

Then the quadratic formula is

$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4(4)(-1)}}{2(4)}$$

$$t = \frac{-8 \pm \sqrt{64 + 16}}{8}$$

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$$t = \frac{-8 \pm 4\sqrt{5}}{8}$$

$$t = \frac{-2 \pm \sqrt{5}}{2}$$

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COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

Solution:

Completing the square gives

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

$$(x - 3)^2 = 4$$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 1, 5$$

2. Fill in the blank with the correct term.

$$x^2 - ____ + \frac{9}{4} = -2 + \frac{9}{4}$$

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Solution:

The blank should be the term $3x$.

■ 3. Complete the square in the following expression, but do not solve.

$$3y^2 - 12y + 3 = 0$$

Solution:

To complete the square, we first write the expression as

$$3y^2 - 12y = -3$$

$$y^2 - 4y = -1$$

Now complete the square as

$$y^2 - 4y + 4 = -1 + 4$$

$$(y - 2)^2 = 3$$

■ 4. Solve for a by completing the square.

$$2a^2 + 8a = -4$$

Solution:

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Completing the square gives

$$a^2 + 4a = -2$$

$$a^2 + 4a + 4 = -2 + 4$$

$$(a+2)^2 = 2$$

$$a + 2 = \pm\sqrt{2}$$

$$a = -2 \pm \sqrt{2}$$

■ 5. What is your first and second step in solving the problem by completing the square?

$$4x^2 - 16x + 28 = 0$$

Solution:

The first step is to move the 28 over to the other side. The second step is to divide everything by 4. These steps could be done in the opposite order, but they are the first two steps you must take before completing the square.

■ 6. Explain when and why completing the square is used for factoring.

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Solution:

Completing the square is used when it's not possible to solve for the roots by factoring.

■ 7. Solve for y by completing the square.

$$3y^2 + 9y = 3$$

Solution:

Completing the square gives

$$3y^2 + 9y = 3$$

$$y^2 + 3y = 1$$

$$y^2 + 3y + \frac{9}{4} = 1 + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$y = -\frac{3 \pm \sqrt{13}}{2}$$

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8. Fill in the blank with the correct term.

$$____ - 4x = 6$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

Solution:

The blank should be filled in with $3x^2$. We can work backwards from the second equation.

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = \frac{22}{9}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{22}{9}$$

$$3x^2 - 4x + \frac{4}{3} = \frac{22}{3}$$

$$3x^2 - 4x = \frac{22}{3} - \frac{4}{3}$$

$$3x^2 - 4x = \frac{18}{3}$$

$$3x^2 - 4x = 6$$

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LONG DIVISION OF POLYNOMIALS

1. Find the quotient.

$$\frac{x^2+2x-1}{x+3}$$

Solution:

Use long division to find the quotient.

$$x-1+\frac{2}{x+3}$$

$$
\begin{array}{r|l}
x+3 & x^2+2x-1 \\
 & -(x^2+3x) \\
\hline
 & -x-1 \\
 & -(-x-3) \\
\hline
 & 2
\end{array}
$$

2. Find the quotient.

$$\frac{2x^3-x^2-4x+5}{x-2}$$

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Solution:

Use long division to find the quotient.

$$
\begin{array}{r}
2x^2+3x+2+\frac{9}{x-2} \\
x-2 \overline{\big)\ 2x^3-x^2-4x+5} \\
-(2x^3-4x^2) \\
\hline
3x^2-4x \\
-(3x^2-6x) \\
\hline
2x+5 \\
-(2x-4) \\
\hline
9
\end{array}
$$

3. Find the quotient.

$$\frac{2x^4+4x^3-x^2+5x-150}{x+4}$$

Solution:

Use long division to find the quotient.

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$$
\begin{array}{r}
2x^3-4x^2+15x-55+\frac{70}{x+4} \\
x+4\overline{)\,2x^4+4x^3-x^2+5x-150} \\
-(2x^4+8x^3) \\
\hline
-4x^3-x^2 \\
-(-4x^3-16x^2) \\
\hline
15x^2+5x \\
-(15x^2+60x) \\
\hline
-55x-150 \\
-(-55x-220) \\
\hline
70
\end{array}
$$

4. Find the quotient.

$$\frac{3x^3 - x^2 - 7x + 5}{x - 1}$$

Solution:

Use long division to find the quotient.

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$$
\begin{array}{r}
3x^2+2x-5 \\
\hline
x-1 \;\big|\; 3x^3-x^2-7x+5 \\
-(3x^3-3x^2) \\
\hline
2x^2-7x \\
-(2x^2-2x) \\
\hline
-5x+5 \\
-(-5x+5) \\
\hline
0
\end{array}
$$

5. Find the quotient.

$$\frac{-x^2+3x+15}{x+5}$$

Solution:

Use long division to find the quotient.

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$$
\begin{array}{r}
-x+8-\dfrac{25}{x+5} \\
x+5 \overline{\big|\, -x^2+3x+15} \\
\underline{-(-x^2-5x)} \\
8x+15 \\
\underline{-(8x+40)} \\
-25
\end{array}
$$

■ 6. Find the quotient.

$$\frac{x^4+x-3}{x-2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^3 and x^2 terms.

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$$x-2\overline{)\,x^4+0x^3+0x^2+x-3}\quad = x^3+2x^2+4x+9+\frac{15}{x-2}$$

$$-(x^4-2x^3)$$

$$2x^3+0x^2$$

$$-(2x^3-4x^2)$$

$$4x^2+x$$

$$-(4x^2-8x)$$

$$9x-3$$

$$-(9x-18)$$

$$15$$

7. Find the quotient.

$$\frac{x^3+6}{x+6}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^2 and x terms.

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$$
\begin{array}{r}
x^2-6x+36-\dfrac{210}{x+6} \\
x+6\,\overline{\big)\,x^3+0x^2+0x+6} \\
-(x^3+6x^2) \\
\hline
-6x^2+0x \\
-(-6x^2-36x) \\
\hline
36x+6 \\
-(36x+216) \\
\hline
-210
\end{array}
$$

8. Find the quotient.

$$\frac{x^2+x}{x-3}$$

Solution:

Use long division to find the quotient. Remember to represent the missing constant term.

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$$
\begin{array}{r}
x+4+\frac{12}{x-3} \\
x-3 \overline{\left| x^2+x+0 \right.} \\
\underline{-(x^2-3x)} \\
4x+0 \\
\underline{-(4x-12)} \\
12
\end{array}
$$

9. Find the quotient.

$$\frac{x^4 - 2x^2}{x - 4}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

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$$x^3+4x^2+14x+56+\frac{224}{x-4}$$

$$x-4 \;\overline{)\; x^4+0x^3-2x^2+0x+0}$$

$$-(x^4-4x^3)$$

$$4x^3-2x^2$$

$$-(4x^3-16x^2)$$

$$14x^2+0x$$

$$-(14x^2-56x)$$

$$56x+0$$

$$-(-56x-224)$$

$$224$$

10. Find the quotient.

$$\frac{-2x^3+8x}{x+2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

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$$
\begin{array}{r}
-2x^2+4x \\
x+2\,\overline{)\,-2x^3+0x^2+8x+0} \\
\underline{-(-2x^3-4x^2)} \\
\qquad\quad 4x^2+8x \\
\qquad\quad \underline{-(4x^2+8x)} \\
\qquad\qquad\quad 0
\end{array}
$$

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